Published by Institute of Physics Publishing for SISSA/ISAS

Received: *March 06, 2002*
Accepted: *April 17, 2002*

Confining strings at high temperature

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Abstract: We show that the high-temperature behaviour of the recently proposed confining strings reproduces exactly the correct large-N QCD result, for a large class of truncations of the long-range interaction between surface elements.

Keywords: Confinement, Bosonic Strings, QCD.

JHEP04(2002)032

*Supported by a Swiss National Science Foundation Fellowship. On leave of absence from I.N.F.N. and University of Perugia, Italy.

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Contents

1. Introduction

Coloured states are not observable as free particles: at large distances the static potential between a quark and an antiquark grows linearly with the distance. Attempting to explain this quark confinement by means of a non-critical string theory dual to QCD remains an open and very important problem [1]. Indeed finding the theory dual to QCD would imply having perturbative control over both the asymptotically free and the confinement regimes of the theory. Despite many efforts, however, the formulation of a consistent string theory description of confinement in 4D non-abelian gauge theories remains an open issue. The Nambu-Goto term can be consistently quantized only in $D = 26$ or $D \leq 1$ and leads to strings with a crumpled world-sheet, inappropriate to describe the expected smooth strings dual to QCD [2]. In order to take into account the bending rigidity of flux tubes, and to cure the problems of the fundamental Nambu-Goto action, a term proportional to the extrinsic curvature of the world-sheet was added to it [3]. This term, however, is infrared-irrelevant and the rigid string thus shares the geometrical problems of the Nambu-Goto action [3].

A new action for confining strings has recently been proposed in [4]. The confining string action is based on an induced string action explicitly derivable for compact QED [5] and for abelian-projected SU(2) [6], and possesses, in its world-sheet formulation, a non-local action with a negative stiffness [5, 7] that can be expressed as a derivative expansion of a long-range interaction between surface elements. In order for the geometrical properties of these strings to be analytically studied, the expansion must be truncated: since the stiffness is negative, a stable truncation must, at least, include a sixth-order term in the derivatives [8]. This string action has many features that make it a good candidate to describe QCD flux tubes, at least in the large-D approximation. In fact this model has an infrared fixed point at zero stiffness [8, 9], corresponding to a tensionless smooth string whose world-sheet has Hausdorff dimension 2, exactly the desired properties to describe QCD flux tubes. The long-range orientational order in this model is due to an antiferromagnetic interaction between normals to the surface [10], a mechanism confirmed by numerical simulations [11]. Moreover, it was shown in [9] that this infared fixed point does not depend on the truncation and is present for all ghost- and tachyon-free truncations, and that the effective theory describing the infrared behaviour is a conformal field theory with central charge $c = 1$.

There is, however another characteristic that any string model describing confinement in QCD must possess, namely the correct high-temperature behaviour. In fact, as shown in [12], the deconfining transition in QCD is due to the condensation of Wilson lines, and the partition function of QCD flux tubes can be continued above the deconfining transition; this high-temperature continuation can be evaluated perturbatively. So, any string theory that is equivalent to QCD *must* reproduce this behaviour. As shown in [13], contrary to previous string models, the high-temperature behaviour of confining strings reproduces exactly the correct large-N QCD result, a *necessary* condition for any string model of confinement.

In [13], the correct high-temperature behaviour of the confining string action was derived for a specific truncation. In this paper we will show that this property is largely independent of the specific truncation we choose, but holds true for all ghost- and tachyon-free truncations.

2. Definition of the model

The model proposed in [8] is

$$S = \int d^2\xi \sqrt{g} g^{ab} \mathcal{D}_a x_\mu \left(t - s\mathcal{D}^2 + \frac{1}{M^2}\mathcal{D}^4 \right) \mathcal{D}_b x_\mu \, , \tag{2.1}$$

where $\mathcal{D}_a$ are covariant derivatives with respect to the induced metric $g_{ab} = \partial_a x_\mu \partial_b x_\mu$ on the surface $\mathbf{x}(\xi_0, \xi_1)$. The first term in the bracket provides a bare surface tension $2t$, while the second accounts for the rigidity, with a stiffness parameter s that can be positive or negative. The third term contains the square of the gradient of the extrinsic curvature matrices, and it suppresses the formation of spikes on the world-sheet. The new mass scale M generates, in the large-D approximation, string tension proportional to M^2, which takes control of the fluctuations where the orientational correlation dies off.

We now want to generalize this model to an arbitrary truncation of the long-range interaction in (2.1). Following [9] we will consider an action of the form:

$$\begin{aligned} S|_n &= \int d^2\xi \sqrt{g} g^{ab} \mathcal{D}_a x_\mu V_n(\mathcal{D}^2) \mathcal{D}_b x_\mu \, , \\ V_n(\mathcal{D}^2) &= t\Lambda^2 + \sum_{k=1}^{2n} \frac{c_k}{\Lambda^{2k-2}} (\mathcal{D}^2)^k \, , \end{aligned} \tag{2.2}$$

where Λ represents the fundamental mass scale in the model, to be identified with the QCD mass scale; c_k are positive numbers, which means that a stable truncation must end with an even $k = 2n$. The model studied in [13] corresponds to $n = 1$.

The parameters c_k are free: the only condition we impose on them is the absence of both tachyons and ghosts in the theory. This requires that the Fourier transform $V_n(p^2)$ have no zeros on the real p^2-axis. The polynomial $V_n(p^2)$ thus has n pairs of complex-conjugate zeros in the complex p^2-plane [9]. To simplify the computation we will also set all coefficients with odd k to zero, $c_{2m+1} = 0$ for $0 \le m \le n-1$. This, however, is no drastic restriction since, as was shown in [9], this is their value at the infrared-stable fixed point anyhow.

We will study this model in the large-D approximation. For this purpose [14] we need to introduce a Lagrange multiplier λ^{ab} that forces the induced metric $\partial_a x_\mu \partial_b x_\mu$ to be equal to the intrinsic metric g_{ab}, extending the action (2.1) to:

$$S|_n \longrightarrow S|_n + \int d^2\xi \sqrt{g} \left[\Lambda^2 \lambda^{ab} (\partial_a x_\mu \partial_b x_\mu - g_{ab})\right] . \tag{2.3}$$

Note that we define here the Lagrange multiplier λ^{ab} as a dimensionless quantity. We parametrize the world-sheet in a Gauss map by $x_\mu(\xi) = (\xi_0, \xi_1, \phi^i(\xi))$, $i = 2, \ldots, D-2$. The value of the periodic coordinate ξ_0 is $-\beta/2 \leq \xi_0 \leq \beta/2$, with $\beta = 1/T$ and T the temperature. The value of ξ_1 is $-R/2 \leq \xi_1 \leq R/2$; $\phi^i(\xi)$ describe the $D-2$ transverse fluctuations. We look for a saddle-point solution with a diagonal metric $g_{ab} = \mathrm{diag}(\rho_0, \rho_1)$, and a Lagrange multiplier of the form $\lambda^{ab} = \mathrm{diag}(\lambda_0/\rho_0, \lambda_1/\rho_1)$. The action then becomes:

$$\begin{aligned}
S|_n &= S|_{n,0} + S|_{n,1} , \\
S|_{n,0} &= A_{\mathrm{ext}} \Lambda^2 \sqrt{\rho_0 \rho_1} \left[t \left(\frac{\rho_0 + \rho_1}{\rho_0 \rho_1} \right) + \lambda_0 \left(\frac{1-\rho_0}{\rho_0} \right) + \lambda_1 \left(\frac{1-\rho_1}{\rho_1} \right) \right] , \\
S|_{n,1} &= \int d^2\xi \sqrt{g} \left[g^{ab} \partial_a \phi^i V_n(\mathcal{D}^2) \partial_b \phi^i + \Lambda^2 \lambda^{ab} \partial_a \phi^i \partial_b \phi^i \right] ,
\end{aligned} \tag{2.4}$$

where $\beta R = A_{\mathrm{ext}}$ is the extrinsic, projected area in coordinate space, and $S|_{n,0}$ is the tree-level contribution. Integrating over the transverse fluctuations in the one-loop term $S|_{n,1}$, we obtain, in the limit $R \to \infty$:

$$S|_{n,1} = \frac{D-2}{2} R \sqrt{\rho_1} \sum_{l=-\infty}^{+\infty} \int \frac{dp_1}{2\pi} \ln \left[(p_1^2 \lambda_1 + \omega_l^2 \lambda_0) \Lambda^2 + p^2 V_n(p^2) \right] , \tag{2.5}$$

where $p^2 = p_1^2 + \omega_l^2$, and $\omega_l = \frac{2\pi}{\beta\sqrt{\rho_0}} l$.

3. High-temperature behaviour

At high temperatures, satisfying

$$\frac{c_{2n}}{\Lambda^{4n-2}} \frac{1}{\beta^{4n}} \gg \Lambda^2 t + \sum_{k=1}^{2n-1} \frac{c_k}{\Lambda^{2k-2}} \frac{1}{\beta^{2k}} , \tag{3.1}$$

with k even, the highest-order term in the derivatives dominates in the one-loop term $S|_{n,1}$ when $n \neq 0$. Using analytic regularization and analytic continuation of the formula $\sum_{n=1}^{\infty} n^{-z} = \zeta(z)$, for the Riemann zeta function, with $\zeta(-1) = -1/12$, we obtain for the $n \neq 0$ contribution:

$$\begin{aligned}
&\frac{D-2}{2} R \sqrt{\rho_1} \sum_{l=-\infty}^{+\infty} \int \frac{dp_1}{2\pi} \ln \frac{c_{2n}}{\Lambda^{4n-2}} \left(\omega_l^2 + p_1^2 \right)^{2n+1} = \\
&= \frac{D-2}{2} \sqrt{\frac{\rho_1}{\rho_0}} (2n+1) 4\pi \frac{R}{\beta} \sum_{l=1}^{+\infty} \sqrt{l^2} = -\frac{D-2}{2} \sqrt{\frac{\rho_1}{\rho_0}} \frac{(2n+1)\pi}{3} \frac{R}{\beta} .
\end{aligned} \tag{3.2}$$

The calculation of the $n=0$ contribution is a little more involved. For $n=0$ we can rewrite $S|_{n,1}$ as:

$$S|_{n,1} = \frac{D-2}{2} R\sqrt{\rho_1} \int \frac{dp_1}{2\pi} \ln\left(p_1^2 \bar{V}_n(p_1^2)\right),$$

with

$$\bar{V}_n(p_1^2) = \left(\Lambda^2(t+\lambda_1) + \sum_{k=1}^{2n} \frac{c_k}{\Lambda^{2k-2}} p_1^{2k}\right), \tag{3.3}$$

and k even. With the simplification we have introduced for the c_k, namely that all coefficients with odd k are zero, and the requirement that the model is ghost- and tachyon-free, all pairs of complex-conjugate zeros of $\bar{V}_n(p_1^2)$ lie on the imaginary axis and we can represent $\bar{V}_n(p_1^2)$ as [9]

$$\frac{\Lambda^{4n-2}}{c_{2n}} \bar{V}_n\left(p_1^2\right) = \prod_{k=1}^{n} \left(p_1^4 + \alpha_k^2 \Lambda^4\right), \tag{3.4}$$

with purely numerical coefficients α_k. The $n=0$ contribution then becomes:

$$\begin{aligned} S|_{n,1} &= \frac{D-2}{2} R\sqrt{\rho_1} \sum_{k=1}^{n} \int \frac{dp_1}{2\pi} \ln\left(p_1^4 + \alpha_k^2 \Lambda^4\right) \\ &= \frac{D-2}{2} R\sqrt{\rho_1} \sum_{k=1}^{n} \int \frac{dp_1}{2\pi} 2\,\mathrm{Re}\ln\left(p_1^4 + i\alpha_k \Lambda^2\right) = \frac{D-2}{2} R\sqrt{\rho_1} \sum_{k=1}^{n} \Lambda\sqrt{2\alpha_k}\,, \end{aligned} \tag{3.5}$$

and we obtain a total action of the form:

$$S_n = S|_{n,0} + \frac{D-2}{2} R\sqrt{\rho_1} \left[\sum_{k=1}^{n} \Lambda\sqrt{2\alpha_k} - \frac{(2n+1)\pi}{3\sqrt{\rho_0}} \frac{1}{\beta}\right]. \tag{3.6}$$

Note that the p_1-independent term in (3.4) must satisfy:

$$\prod_{k=1}^{n} \alpha_k^2 \Lambda^4 = \frac{\Lambda^{4n}}{c_{2n}} (t+\lambda_1)\,. \tag{3.7}$$

Following [9], we assume that all α_k are equal so that (3.7) implies:

$$\alpha_k^2 = (t+\lambda_1)^{1/n} \alpha^2\,, \qquad \alpha = \left(\frac{1}{c_{2n}}\right)^{1/2n}. \tag{3.8}$$

With this assumption we obtain the four large-D gap equations:

$$\frac{1-\rho_0}{\rho_0} = 0\,, \tag{3.9}$$

$$\frac{1}{\rho_1} = 1 - \frac{D-2}{2} \frac{1}{4\beta\Lambda} \sqrt{2\alpha} (\lambda_1 + t)^{1/4n-1}\,, \tag{3.10}$$

$$\left[\frac{1}{2}(t-\lambda_1) + \frac{1}{2\rho_1}(\lambda_1 + t) - t - \lambda_0\right] + \frac{D-2}{2} \frac{(2n+1)\pi}{6\beta^2\Lambda^2} = 0\,, \tag{3.11}$$

$$(t-\lambda_1) - \frac{1}{\rho_1}(\lambda_1 + t) + \frac{D-2}{2} \frac{1}{\beta\Lambda} \left[\sqrt{2\alpha}\, n\, (\lambda_1 + t)^{1/4n} - \frac{\pi(2n+1)}{3\beta\Lambda}\right] = 0\,. \tag{3.12}$$

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Inserting (3.12) and (3.9) into (3.6) and using $\rho_0 = 1$ from (3.9) we obtain a simplified form of the effective action:

$$S^{\mathrm{eff}} = A_{\mathrm{ext}} \Lambda^2 \mathcal{T} \sqrt{\frac{1}{\rho_1}}\,, \tag{3.13}$$

with $\mathcal{T} = 2(\lambda_1 + t)$ representing the physical string tension.

Without loss of generality we now set

$$\sqrt{2\alpha} = \gamma (\lambda_1 + t)^{-1/4n+1/2}\,. \tag{3.14}$$

Inserting (3.11) into (3.12) we then obtain an equation for $(\lambda_1 + t)$ alone:

$$(\lambda_1 + t) - \frac{D-2}{2}\frac{4n+1}{8\beta\Lambda}\gamma\,(\lambda_1+t)^{1/2} + \frac{D-2}{2}\frac{2n+1}{3}\frac{\pi}{2\beta^2\Lambda^2} - t = 0\,. \tag{3.15}$$

This equation has two solutions:

$$\begin{aligned}(\lambda_1+t)^{1/2} &= \frac{D-2}{2}\frac{4n+1}{16\beta\Lambda}\times \\ &\times \left[1 \pm \sqrt{1 - \frac{2n+1}{3}\frac{128\pi}{(4n+1)^2\gamma^2\frac{D-2}{2}} + \frac{256\ t\ \beta^2\ \Lambda^2}{(4n+1)^2\gamma^2\left(\frac{D-2}{2}\right)^2}}\,\right].\end{aligned} \tag{3.16}$$

When

$$\gamma > \left(\frac{D-2}{2}\right)^{-1/2}\frac{8}{(4n+1)}\sqrt{\frac{2(2n+1)\pi}{3}}\,, \tag{3.17}$$

the solutions (3.16) are both real, independently of β. As in [13], for the solution with the plus sign, $1/\rho_1$ is always positive (note that $n \geq 1$):

$$\frac{1}{\rho_1} = 1 - \frac{4}{(4n+1)\left[1 + \sqrt{1 - \frac{2n+1}{3}\frac{128\pi}{(4n+1)^2\gamma^2\frac{D-2}{2}} + \frac{256\ t\ \beta^2\ \Lambda^2}{(4n+1)^2\gamma^2\left(\frac{D-2}{2}\right)^2}}\,\right]}\,. \tag{3.18}$$

The square of the free energy $F^2(\beta) \equiv S^2_{\mathrm{Eff}}/R^2$ is thus positive, since $(\lambda_1 + t)$ is real:

$$\begin{aligned}F^2(\beta) &= \frac{1}{\beta^2}\left(\frac{D-2}{2}\frac{(4n+1)}{16}\right)^4 \times \\ &\times \left[1 - \sqrt{1 - \frac{2n+1}{3}\frac{128\pi}{(4n+1)^2\gamma^2\frac{D-2}{2}} + \frac{256\ t\ \beta^2\ \Lambda^2}{(4n+1)^2\gamma^2\left(\frac{D-2}{2}\right)^2}}\,\right]^4 \times \\ &\times \left[1 - \frac{4}{(4n+1)\left[1 + \sqrt{1 - \frac{2n+1}{3}\frac{128\pi}{(4n+1)^2\gamma^2\frac{D-2}{2}} + \frac{256\ t\ \beta^2\ \Lambda^2}{(4n+1)^2\gamma^2\left(\frac{D-2}{2}\right)^2}}\,\right]}\right].\end{aligned} \tag{3.19}$$

As for the rigid string [16], the high-temperature behaviour is the same as in QCD, but the sign is wrong. The crucial difference with respect to the rigid string case is that (3.19) is real, while the squared free energy for the rigid string is imaginary, signalling an instability in the model [16]. If we now look at the behaviour of ρ_1 at low temperatures, below the deconfining transition [9], we see that $1/\rho_1$ is positive. The deconfining transition is indeed determined by the vanishing of $1/\rho_1$ at $\beta = \beta_{\rm dec}$. In the case of (3.18), this means that $1/\rho_1$ is positive below the Hagedorn transition, touches zero at $\beta_{\rm dec}$ and remains positive above it. Exactly the same will happen also for F^2, which is positive below $\beta_{\rm dec}$, touches zero at $\beta_{\rm dec}$ and remains positive above it. This solution thus describes an unphysical "mirror" of the low-temperature behaviour of the confining string, without a real deconfining Hagedorn transition. For this reason we discard it.

We will now concentrate on the solution of (3.16) with the minus sign. In this case we have

$$\frac{1}{\rho_1} = 1 - \frac{4}{(4n+1) - (4n+1)\sqrt{1 - \frac{2n+1}{3}\frac{128\pi}{(4n+1)^2\gamma^2\frac{D-2}{2}} + \frac{256\ t\ \beta^2\ \Lambda^2}{(4n+1)^2\gamma^2\left(\frac{D-2}{2}\right)^2}}}\,. \tag{3.20}$$

If

$$\gamma > \left(\frac{D-2}{2}\right)^{-1/2} 4\sqrt{\frac{2n+1}{4n-1}}\sqrt{\frac{\pi}{3}}\,, \tag{3.21}$$

$1/\rho_1$ is negative independently, of β. Note that, when (3.21) is satisfied, then also (3.17) is automatically satisfied. We will restrict to this range of values for γ. In this case, as in [13], the physical string tension is real and proportional to $1/\beta^2$ and $1/\rho_1$ is negative, giving:

$$\begin{aligned} F^2(\beta) \;=\; & -\frac{1}{\beta^2}\left(\frac{D-2}{2}\frac{(4n+1)}{16}\right)^4 \times \\ & \times \left[1 - \sqrt{1 - \frac{2n+1}{3}\frac{128\pi}{(4n+1)^2\gamma^2\frac{D-2}{2}} + \frac{256\ t\ \beta^2\ \Lambda^2}{(4n+1)^2\gamma^2\left(\frac{D-2}{2}\right)^2}}\right]^4 \times \\ & \times \left[\frac{4}{(4n+1)\left[1 - \sqrt{1 - \frac{2n+1}{3}\frac{128\pi}{(4n+1)^2\gamma^2\frac{D-2}{2}} + \frac{256\ t\ \beta^2\ \Lambda^2}{(4n+1)^2\gamma^2\left(\frac{D-2}{2}\right)^2}} - 1\right]}\right]. \end{aligned} \tag{3.22}$$

In the range defined by (3.21), this is *negative*. Note, moreover, that for $n = 1$ we exactly reproduce the result found in [13]. Since the sign of λ_1 does not change at high temperatures, the field x_μ is not unstable. In the rigid string case [16], instead, the change of sign of λ_1 gives rise to a world-sheet instability.

Let us now compare with the large-N QCD result [12]:

$$F^2(\beta)_{\rm QCD} = -\frac{2g^2(\beta)N}{\pi^2\beta^2}\,, \tag{3.23}$$

where $g^2(\beta)$ is the QCD coupling constant. To this end we first simplify our result by considering

$$\gamma \gg \left(\frac{D-2}{2}\right)^{-1/2}\frac{8}{(4n+1)}\sqrt{\frac{2(2n+1)\pi}{3}}\,.$$

Moreover we ask that

$$t\beta^2 \ll \frac{D-2}{2}\,, \tag{3.24}$$

so that we can ignore the last term in the square root in (3.22). Note that this condition is compatible with the high-temperature approximation (3.1). In this case (3.22) reduces to

$$F^2(\beta) = -\frac{1}{\beta^2}\frac{4\pi^3}{125}\frac{(2n+1)^3}{(4n+1)^2}\frac{D-2}{\gamma^2}\,. \tag{3.25}$$

This corresponds *exactly* to the QCD result (3.23), with the identifications

$$g^2 \propto \frac{1}{\gamma^2}\,,$$
$$N \propto D-2\,.$$

The weak β-dependence of the QCD coupling $g^2(\beta)$ can be accommodated in the parameter γ. As one would expect for an asymptotic free theory [17], our high-temperature result is valid at large values of γ, i.e. small values of g^2. As first noted in [13] there is an interesting relation between the order of the gauge group and the number of transverse space-time dimensions.

Together with the absence of crumpling, this result is a very strong indication that non-local interactions between surface elements are crucial to describe the world-sheet of QCD flux tubes. Investigations of the fundamental string model in which these interactions are mediated by an antisymmetric tensor field are in progress.

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